Math 45 SSM 2/e 5.5 Dividing Polynomials

Objectives: 1) Divide a polynomial by a monomial by dividing each term
2) Divide a polynomial by a polynomial using long division.

Divide

yes (1) $\frac{9x^3 - 21x^2}{3x}$

step 0: recognize poly ÷ mono

step 1: divide each term by monomial.

step 2: reduce each fraction by dividing monomials

step 3: write with positive exponents.

$= \frac{9x^3}{3x} - \frac{21x^2}{3x}$

$= \boxed{3x^2 - 7x}$

No (2) $(12p^4 + 24p^3 + 4p^2) \div (4p^2)$ poly ÷ mono

$= \frac{12p^4}{4p^2} + \frac{24p^3}{4p^2} + \frac{4p^2}{4p^2}$ divide each term

$= \boxed{3p^3 + 6p + 1}$ reduce

yes (3) $\frac{8a^2b^2 - 6a^2b + 5ab^2}{2a^2b^2}$ poly ÷ mono

$= \frac{8a^2b^2}{2a^2b^2} - \frac{6a^2b}{2a^2b^2} + \frac{5ab^2}{2a^2b^2}$ divide each term

$= 4 - \frac{3}{b} + \frac{5}{2a}$ reduce

$= \boxed{4 - \frac{3}{b} + \frac{5}{2a}}$

yes (4) Divide 2348 by 53 using long division. (This will be our model problem.)

$$53\overline{)2348}\quad 44$$
$$-212$$
$$228$$
$$-212$$
$$16$$

$\boxed{44\frac{16}{53}}$

$\frac{1}{53} \times 6 = 318$ $\quad \frac{1}{53} \times 5 = 265$ $\quad \frac{1}{53} \times 4 = 212$

If you are in the habit (or were taught) to do the subtraction in your head, please do not try to do the polynomial subtraction in your head.

No (5) Divide $x^2+10x+21$ by $x+8$

$x+8 \overline{)\ x^2+10x+21}$ ← step 1 Write both divisor (outside) and dividend in standard form – highest exp first.

step 2: if necessary, write 0 place-holders for any missing exp. (Does not apply to this problem.)

$$\begin{array}{r} x \\ x+8 \overline{)\ x^2+10x+21} \\ x^2+8x \\ \hline \end{array}$$

step 3: Divide first term of dividend by first term of divisor to get first term of quotient

$\frac{x^2}{x} = x$

step 4: Multiply result from step 3 by divisor using distribute

$x(x+8) = x^2+8x$

$$\begin{array}{r} x+2 \\ x+8 \overline{)\ x^2+10x+21} \\ +\ \underline{-x^2-8x} \\ 0\ +2x+21 \\ \underline{\quad 2x+16} \end{array}$$

step 5: subtract dist neg to all terms & add. vertical method.

step 6: bring down next term of dividend

$$\begin{array}{r} x+2 \\ x+8 \overline{)\ x^2+10x+21} \\ \underline{-x^2-8x} \\ 2x+21 \\ \underline{-2x-16} \\ 5 \end{array}$$

step 7: repeat steps 3–6.

$\frac{2x}{x} = 2$

$2(x+8) = 2x+16$

step 8: Write result with remainder

$$x+2+\frac{5}{x+8}$$

Math 45 SSM 2/e 5.5 p.3

No ⑥ Divide $6x^2+9x-10$ by $2x-1$ poly ÷ poly

$$\begin{array}{r} 3x+6 \\ 2x-1\ \overline{)\ 6x^2+9x-10} \\ -6x^2+3x \\ \hline 12x-10 \\ -12x+6 \\ \hline -4 \end{array}$$

$3x+6+\frac{-4}{2x-1}$

Scratch work:

$\frac{6x^2}{2x}=3x$

$3x(2x-1)=6x^2-3x$

$\frac{12x}{2x}=6$

$6(2x-1)=12x-6$

No ⑦ $8-9x+2x^2+12x^3+5x^5$ by x^2+3. poly ÷ poly

$$\begin{array}{r} 5x^3 \quad -3x+2 \\ x^2+0x+3\ \overline{)\ 5x^5+0x^4+12x^3+2x^2-9x+8} \\ -5x^5 \quad -15x^3 \\ \hline 0x^4-3x^3+2x^2-9x \\ +3x^3 \quad +9x \\ \hline 2x^2+0x+8 \\ -2x^2 \quad -6 \\ \hline 2 \end{array}$$

$5x^3-3x+2+\frac{2}{x^2+3}$

Scratch:

$\frac{5x^5}{x^2}=5x^3$

$5x^3(x^2+3)=5x^5+15x^3$

$\frac{-3x^3}{x^2}=-3x$

$-3x(x^2+3)=-3x^3-9x$

$\frac{2x^2}{x^2}=2$

$2(x^2+3)=2x^2+6$

Note: Remove placeholders when writing final answers.

Note: Placeholders in divisor are optional.

Note: If you write terms of quotient above like terms, you'll be done when you get to the end of the division bar.

↳ You'll also know how many terms to bring down.

Note: Officially, you are done when the degree of the remainder is less (or equal) than degree of divisor.

⑦ alternate

$(8 - 9x + 2x^2 + 12x^4 + 5x^5)$ by (x^2+3)

Poly ÷ Poly (long division)

$$
\begin{array}{r}
5x^3 + 12x^2 - 15x - 34 \\
x^2+3 \;\overline{)\; 5x^5 + 12x^4 + 0x^3 + 2x^2 - 9x + 8} \\
\underline{-5x^5 \qquad +15x^3} \\
12x^4 - 15x^3 + 2x^2 \\
\underline{-12x^4 \qquad +36x^2} \\
-15x^3 - 34x^2 - 9x \\
\underline{+15x^3 \qquad +45x} \\
-34x^2 + 36x + 8 \\
\underline{+34x^2 \qquad +102} \\
36x + 110
\end{array}
$$

$\frac{5x^5}{x^2} = 5x^3$

$5x^3(x^2+3)$

$\frac{12x^4}{x^2} = 12x^2$

$12x^2(x^2+3)$

$\frac{-15x^3}{x^2} = -15x$

$-15x(x^2+3)$

$\frac{-34x^2}{x^2} = -34$

$-34(x^2+3)$

Answer $\boxed{5x^3+12x^2-15x-34+\frac{36x+110}{x^2+3}}$

(8) $\dfrac{64x^6 - 27}{4x^2 - 3}$ poly ÷ poly

NO

$$4x^2+0x-3 \;\overline{)\; 64x^6+0x^5+0x^4+0x^3+0x^2+0x-27}$$

$16x^4 \quad +12x^2 \quad +9$

$-64x^6 \quad +48x^4$

$0x^5+48x^4+0x^3+0x^2$

$-48x^4 \quad +36x^2$

$0x^3+36x^2+0x-27$

$-36x^2 \quad +27$

$0.$

$\frac{64x^6}{4x^2} = 16x^4$

$\frac{48x^4}{4x^2} = 12x^2$

$\frac{36x^2}{4x^2} = 9$

$\boxed{16x^4+12x^2+9}$

(9) $\dfrac{25a^3b^2c + 10a^2bc^3}{-5a^4b^2c}$ poly ÷ mono

NO

$$= \frac{25a^3b^2c}{-5a^4b^2c} + \frac{10a^2bc^3}{-5a^4b^2c}$$

$$= \frac{-5}{a} + \frac{-2c^2}{a^2b}$$

$$= \boxed{\frac{-5}{a} - \frac{2c^2}{a^2b}}$$

(10) $\dfrac{-16x + 70 + x^2}{-9 + x}$

NO

$$x-9 \;\overline{)\; x^2-16x+70}$$

$x - 7$

$-x^2+9x$

$-7x+70$

$+7x+-63$

7

write both divisor + dividend in standard form

$\frac{x^2}{x} = x$

$\frac{-7x}{x} = -7$

$\boxed{x-7+\frac{7}{x-9}}$

(11) $\dfrac{18+x^4-9x^2+3x^3-9x}{x^2-3}$ poly ÷ poly

$$\begin{array}{r} x^2+3x-6 \\ x^2+0x-3\,\overline{)\,x^4+3x^3-9x^2-9x+18} \\ -x^4+0x^3+3x^2 \\ \hline 3x^3-6x^2-9x \\ -3x^3+0x^2+9x \\ \hline -6x^2+0x+18 \\ +6x^2+0x+78 \\ \hline 0 \end{array}$$

$\frac{x^4}{x^2}=x^2$

$\frac{3x^3}{x^2}=3x$

$\frac{-6x^2}{x^2}=-6$

$\boxed{x^2+3x-6}$

NO (12) $\dfrac{4+7x^2-3x^4+6x^3}{2x^2}$ poly ÷ mono

$=-\frac{3x^4}{2x^2}+\frac{6x^3}{2x^2}+\frac{7x^2}{2x^2}+\frac{4}{2x^2}$ write in standard form; divide each term

$=\boxed{\frac{-3x^2}{2}+3x+\frac{7}{2}+\frac{2}{x^2}}$

NO (13) $\dfrac{4x^2+5}{1+2x}$ poly ÷ poly

$$\begin{array}{r} 2x-1 \\ 2x+1\,\overline{)\,4x^2+0x+5} \\ -4x^2+2x \\ \hline -2x+5 \\ +2x+1 \\ \hline 6 \end{array}$$

$\frac{4x^2}{2x}=2x$

$\frac{-2x}{2x}=-1$

$\boxed{2x-1+\frac{6}{2x+1}}$